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Interaction of Relativistic Electron Beams with Fusion Target Blow-off Plasmas

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The relativistic Boltzmann equation with a Fokker-Planck collision term is solved with the
assumption that elastic scattering characterizes the shortest time and length scales of the beam-
plasma system. Explicit one-dimensional solutions containing the self-consistent return-current
electric field show that although range-shortening occurs, the primary effect of the electric
field is to prevent high-current-density beams from penetrating into the dense-plasma region.

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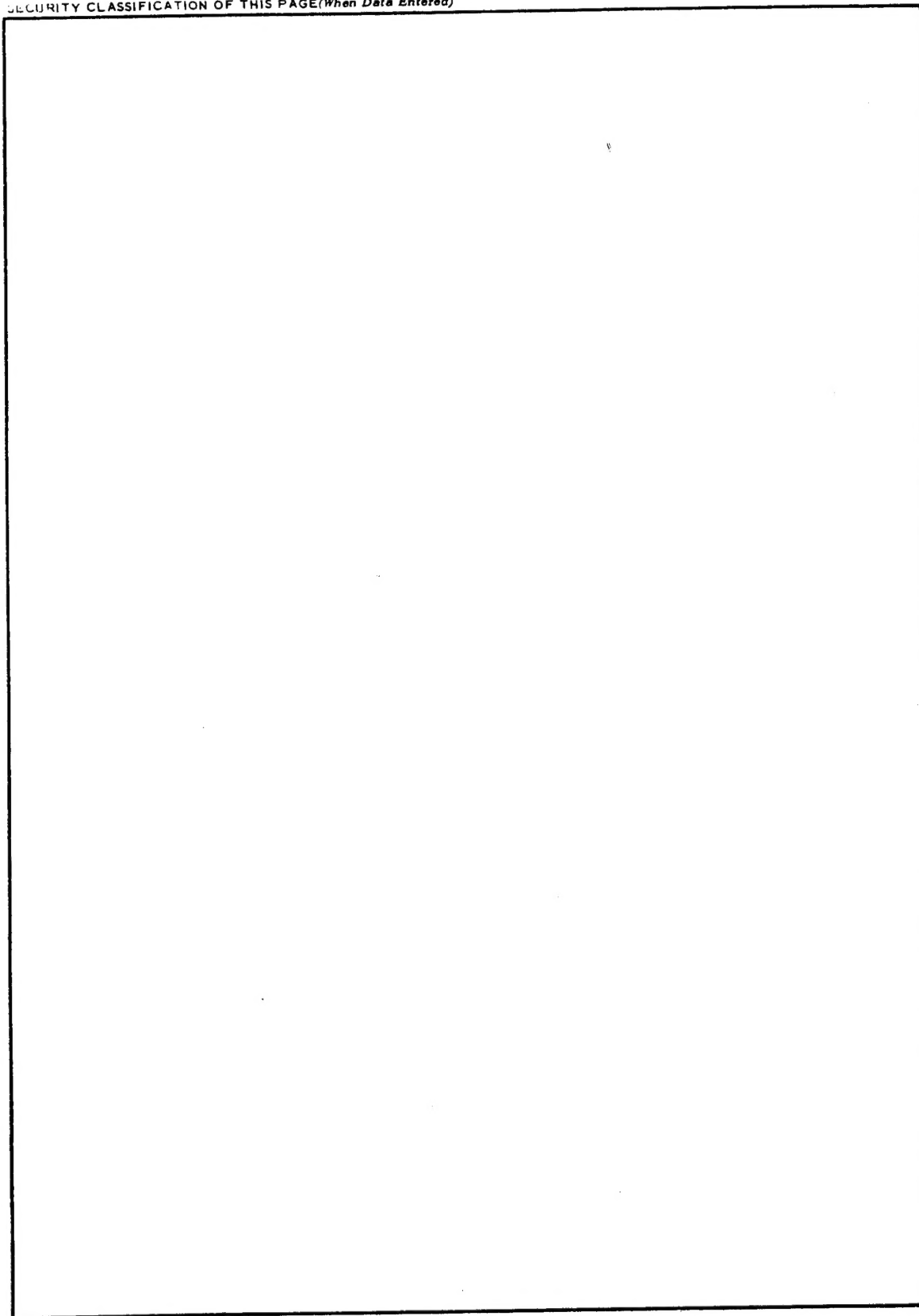
INTERACTION OF RELATIVISTIC ELECTRON BEAMS
WITH FUSION TARGET BLOW-OFF PLASMAS

The development of high-power pulse generators has created interest in the use of relativistic electron beams to compress and heat small masses of deuterium and tritium to fusion in a manner similar to that proposed using lasers.[1,2] In order to optimize thermonuclear yield, fusion-pellet designs employ thin shells of high-atomic-number material in which the beam deposits energy.[3,4] Hydrodynamic modeling of pellet implosion has neglected both the effects of the electromagnetic field and scattering collisions in the beam-heated plasma blown off of the shell.[1,3] This work presents a formalism which allows one to determine the character of beam deposition in the high-atomic-number plasma and shell when these effects are included. The Boltzmann equation describing the beam with a relativistically-correct Fokker-Planck collision term is solved with the assumption that the elastic-scattering collision time is the shortest characteristic time of the system. (This approximation is valid for cases of interest: the interaction of a 1-3 MeV electron beam of about 10 nsec duration with initially-solid shells of heavy material.) One-dimensional solutions are then calculated in two limiting cases of interest. From these, the energy-deposition profile and efficiency of energy coupling from beam to plasma are determined. Finally, the constraints placed on beams for fusion in light of the present analysis are discussed.

The equation describing the momentum distribution function of relativistic electrons interacting with a cold, high-atomic-number plasma may be written[5]

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m\gamma}\cdot\nabla f - e\left(\vec{E} + \frac{\vec{p}\times\vec{B}}{m\gamma}\right)\cdot\nabla_p f = \nabla_p\cdot[\upsilon_S(p)(p^2\vec{I} - \vec{p}\vec{p})\cdot\nabla_p f] + \nabla_p\cdot[\upsilon_E(p)\vec{p}f] \qquad (1)$$

where $\gamma^2 = 1 + p^2/(mc)^2$. The quantities υ_S and υ_E are energy-dependent

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scattering and energy-loss frequencies

$$\upsilon_S = \Omega_S \gamma / (\gamma^2 - 1)^{3/2}; \qquad\qquad \upsilon_E = \epsilon \gamma \upsilon_S \qquad\qquad (2)$$

where

$$\Omega_S = 2\pi n_i r_o^2 c (Z^2 + Z) \ln\Lambda; \qquad \epsilon = \frac{2}{Z+1} .$$

In the above, n_i is the plasma ion density, r_o is the classical electron radius, c is the velocity of light, Z is the plasma atomic number, and $\ln\Lambda$ is in the range 10-20 for plasmas of interest.[5] We limit consideration to plasmas for which $\epsilon \ll 1$.

Treating ϵ as second order in υ_S^{-1}, the terms of Eq. (1) are ordered according to

$$0(\upsilon_S^{-2}):0(\upsilon_S^{-1}):0(\upsilon_S^{-1}) = 0(1):0(\upsilon_S^{-2}).$$

When f is expanded in powers of υ_S^{-1}

$$f = f_o + f_1 + \cdots ,$$

Eq. (1) can be iteratively solved starting with $0(1)$ terms. The solution correct to second order is given by

$$f_o = f_o (p, \vec{x}, t) \qquad\qquad (3)$$

that is, f_o is isotropic in momentum space and

$$f_1 = \vec{A} \cdot \vec{p} \qquad\qquad (4)$$

$$f_2 = \frac{1}{6\upsilon_S} \left(\frac{e\vec{E}}{p} \frac{\partial \vec{A}}{\partial p} - \frac{1}{m\gamma} \nabla\vec{A} \right) : \vec{p}\vec{p} + \frac{e}{2\upsilon_S m\gamma} (\vec{B}\times\vec{A}) \cdot \vec{p} \qquad\qquad (5)$$

where

$$\vec{A} = \frac{1}{2\upsilon_S} \left(\frac{e\vec{E}}{p} \frac{\partial f_o}{\partial p} - \frac{1}{m\gamma} \nabla f_o \right).$$ (6)

The equation governing f_o is obtained by setting secular terms in the second-order equation equal to zero

$$\frac{\partial f_o}{\partial t} + \frac{p^2}{3m\gamma} \nabla\cdot\vec{A} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^3(\upsilon_E f_o + \frac{1}{3} e\vec{E}\cdot\vec{A}) \right].$$ (7)

In this work, moments of f which yield the beam-electron and energy fluxes are of interest. To lowest significant order, these quantities are determined from

$$\vec{\Phi} = \int \frac{\vec{p}}{m\gamma} f_1 d^3p = \int \frac{\overrightarrow{pp}}{m\gamma} \cdot\vec{A} \, d^3p$$ (8)

$$\vec{q} = \int mc^2(\gamma-1)\frac{\vec{p}}{m\gamma} f_1 d^3p = mc^2 \int (\gamma-1)\frac{\overrightarrow{pp}}{m\gamma} \cdot\vec{A} \, d^3p .$$ (9)

Taking the divergence of $\vec{\Phi}$, substituting from Eq. (7) for $\nabla\cdot\vec{A}$, and integrating by parts yields

$$\nabla\cdot\vec{\Phi} = -4\pi \left(p^3\upsilon_E f_o \right)_p = 0$$ (10)

Lack of particle conservation is due to beam electrons, slowed by dynamic friction to very low energies, merging with the thermal-electron backround. These are not accounted for in f since Eq. (1) describes only the distribution of super-thermal electrons.

The rate at which energy is transferred from the beam to a unit volume of plasma is obtained by taking the divergence of Eq. (9). In the steady state,

$$Q = -\nabla \cdot \vec{q} = e\vec{E} \cdot \vec{\Phi} + 4\pi \int_0^\infty \frac{p^4 \upsilon_E}{m\gamma} f_0 dp. \qquad (11)$$

Steady-state solutions of f in one dimension are now considered. The plane $x = 0$ is chosen to divide a semi-infinite, uniform plasma occupying the region $x > 0$ from a vacuum. A monoenergetic, well-collimated beam of rela-tivistic electrons (particle flux Φ_i) propagating in the vacuum is normally incident on the plasma. The plasma is assumed to be sufficiently conductive to exclude the vacuum magnetic field associated with the incident beam for times of interest.[1] Thus, a plasma return current $j = e\Phi$ of thermal electrons must flow towards the vacuum-plasma interface. The electric field associated with this current is

$$E = \eta j = \eta e\Phi \qquad (12)$$

where η is the plasma resistivity.[6] Plasma electrons which reach the interface are assumed to flow as $\nabla P \times \vec{B}$ surface currents along the $x = 0$ plane. This planar model approximates a relativistic electron beam inci-dent on a spherical shell of high-atomic-number material for a shell thickness small compared to the radius of the sphere.

The beam-electron distribution function in the plasma can now be written

$$f = f_0(p,x) + \mu p A - \frac{\mu^2}{2\upsilon_S} \left[eEA + \frac{1}{p^2} \frac{\partial}{\partial p} (p^3 \upsilon_E f_0) \right] \qquad (13)$$

correct to second order. The second-order term has been simplified using the secular equation

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$$\frac{\partial A}{\partial x} = \frac{m\gamma}{p^4} \frac{\partial}{\partial p} \left[p^3 (3\upsilon_E f_o + eEA) \right] \tag{14}$$

where

$$A = \frac{1}{2\upsilon_S} \left(\frac{eE}{p} \frac{\partial}{\partial p} - \frac{1}{m\gamma} \frac{\partial}{\partial x} \right) f_o \tag{15}$$

and μ is the cosine of the polar angle in a spherical momentum-space geometry with polar axis along x.

The beam-electron flux is given by

$$\Phi(x) = 2\pi \int_{-1}^{+1} d\mu \, \mu^2 \int_0^\infty dp \, \frac{p^4 A}{m\gamma} \tag{16}$$

When $\upsilon_E f_o$ and eEA are comparable in magnitude, Eqs. (12) - (16) must be solved simultaneously using numerical techniques. Two limiting cases of interest which are amenable to analytical solution are now discussed.

When the plasma density is high and the beam-current density is low, dynamic friction dominates over energy loss to the electric field. Neglecting electric-field terms in Eq. (14) leads to the solution[7,8]

$$\gamma^2 f_o = \frac{C \, x}{\tau^{3/2}} \exp\left(-\frac{x^2}{4a^2 \tau} \right) \tag{17}$$

for an incident monoenergetic beam with $\gamma = \gamma_o$. In Eq. (17),

$$\tau = \int_\gamma^{\gamma_o} \frac{(\gamma^2-1)^3 d\gamma}{\gamma^4} = \Gamma(\gamma_o) - \Gamma(\gamma), \tag{18}$$

$$\Gamma(\gamma) = \frac{(\gamma^2+1)(\gamma^4-10\gamma^2+1)}{3\gamma^3}, \tag{19}$$

$a^2 = \frac{c^2}{6\epsilon\Omega_S^2}$, and C is constant. The corresponding electron flux is

$$\Phi(x) = \Phi_o \exp\left(-\frac{x^2}{4a^2\tau_1}\right) \tag{20}$$

where $\tau_1 = \Gamma(\gamma_o) - \Gamma(1)$, and $\Phi_o = \Phi(o)$. The volume heating rate when $E = 0$ may be written

$$Q_C(x) = \frac{mc^2\Phi_o \tau_1^{1/2} x}{2a^2} \int_1^{\gamma_o} \exp\left(-\frac{x^2}{4a^2\tau}\right) \frac{d\gamma}{\tau^{3/2}} \tag{21}$$

The variation of Q_C with x is shown in Fig. 1.

The transmission coefficient of beam current at the interface can be estimated by equating Φ_i with the positive-going beam current there.[7]

$$\Phi_i = 2\pi \int_o^1 d\mu \int_o^\infty dp \ p^2 f \frac{p\mu}{m\gamma} = \pi \int_o^\infty \frac{p^3 f_o(p,0)dp}{m\gamma} + \frac{1}{2}\Phi_o \tag{22}$$

correct to first order. With f_o given by Eq. (17), the transmission coefficient is

$$T_C = \Phi_o/\Phi_i = \left[\frac{1}{2} + \left(\frac{3\pi}{8\epsilon}\right)^{\frac{1}{2}} \frac{\gamma_o^2 \tau_1^{1/2}}{(\gamma_o^2-1)^2}\right]^{-1} \tag{23}$$

For $\gamma_o = 3$, T_C ranges from .75 for aluminum to .40 for gold.

When the beam current is high and the plasma density is sufficiently low, dynamic friction can be neglected in comparison to electric-field slowing down. In that case, Eq. (10) predicts that Φ, and therefore E are constant. The solution of Eq. (14) to first order with $v_E = 0$ for an incident monoenergetic beam is

$$f = f_o + \mu p A = \left\{\alpha-\beta\left[K(\gamma^*)-K(\gamma_o)-\frac{\mathcal{E}\mu}{\gamma^{*2}-1}\right]\right\} \delta(\gamma-\gamma^*) \tag{24}$$

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where α and β are constant, $\mathcal{E} = eE/(mc\Omega_S)$,

$$K(\gamma) = \frac{\gamma(\gamma^2 + 1)}{4(\gamma^2-1)^2} + \frac{1}{8} \ln\left(\frac{\gamma-1}{\gamma+1}\right),$$ (25)

and $\gamma^*(x) = \gamma_o - eEx/(mc^2)$. The quantity β is determined by substitution into Eq. (16).

$$\Phi_o = \frac{4\pi}{3} m^3 c^4 \mathcal{E} \beta$$ (26)

The heating rate in the absence of dynamic friction is given by Eq. (11).

$$Q_E = eE\Phi_o \qquad ; \qquad \gamma^*{\geq}1.$$ (27)

Using Eq. (22), the transmission coefficient is

$$T_E = \frac{4\mathcal{E}\beta/3}{(\gamma_o^2-1)\alpha + 2\mathcal{E}\beta/3}$$ (28)

A reasonable boundary condition for large x is needed to determine α. Since υ_E increases with x according to Eq. (2) and the definition of γ^*, dynamic friction must dominate over the electric field for sufficiently-large x. The values of γ^* and x at which dynamic friction becomes important can be estimated by equating the two terms on the right side of Eq. (14)

$$3\upsilon_e f_o = eEA \qquad @ \quad \gamma^* = \gamma_c = \gamma_o - eEx_c/(mc^2)$$

or

$$\mathcal{E}^2\beta = 3\epsilon\gamma_c^2 \left\{ \alpha - \beta[K(\gamma_c) - K(\gamma_o)] \right\}$$ (29)

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The region $x > x_c$ represents a strong absorber of slowed-down electrons. It is then reasonable to set the negative-going current at $x = x_c$ equal to zero. To first order,

$$\Phi_R(x_c) = 2\pi \int_{-1}^{0} \mu d\mu \int_{0}^{\infty} \frac{p^3 f_o(p,x_c)}{m\gamma} \, dp + \frac{1}{2} \Phi_o = 0$$

or

$$\alpha = \beta \left[\frac{2}{3} \frac{\mathcal{E}}{\gamma_c^2 - 1} + K(\gamma_c) - K(\gamma_o) \right]. \tag{30}$$

Simultaneous solution of Eqs. (29) and (30) yield γ_c and α. The resulting variation of transmission with electric field is shown in Fig. 2 for $\gamma_o = 3$. The electric field is related to the penetrating beam current by Eq. (12).

$$e\Phi_o = 8.4 \times 10^{-17} (Z + 1) n_i \theta^{3/2} \mathcal{E} \qquad \frac{A}{cm^2} \tag{31}$$

Here, θ is the plasma electron temperature in eV.

The above calculations show that although the electric field reduces the beam transmission into the plasma, it does increase the volume heating rate above that due to dynamic friction alone. Its importance to heating can be estimated by comparing the heating rates Q_E and Q_C averaged over x. For $\gamma_o = 3$, $Q_E/\bar{Q}_C \approx \mathcal{E}/\epsilon^{\frac{1}{2}}$, so that electric-field heating dominates when $\mathcal{E} \geqslant \epsilon^{\frac{1}{2}}$. The region indicated by this inequality is shown in Fig. 2. It is seen that electric-field (or return-current) heating is important only when $\mathcal{E}$ is large enough to reflect all but a small portion of the incident beam. Thus, in order to maximize energy transfer from beam to plasma, the plasma should be prepared in a manner which keeps $\mathcal{E}$ small everywhere. In terms

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of the incident current $J_i = e\Phi_o/T_E$, this condition takes the form

$$J_i \leq 10^{-15} n_i \theta^{3/2} \qquad \frac{A}{cm^2} \qquad (32)$$

For incident currents of 10^9 A/cm^2 and 1 keV temperatures,[1] Eq. (32) suggests that poor penetration occurs into plasmas with ion densities less than 10^{19} cm^{-3}. If, during pellet irradiation, the blow-off has sufficient time to expand to a low density region of large extent, poor coupling of the beam to the dense-plasma or solid portion of the shell is predicted. However, if the beam is not strongly scattered in the low-density blow-off, high transmission to, and collisional heating in the dense plasma region can occur. For blow-off plasmas obeying fluid-equation similarity solutions,[2] the condition that the beam not be strongly scattered in the region $n_i \leq 10^{15} J_i \theta^{-3/2}$ is

$$J_i \tau < 10^2 \gamma_o^2 \theta/Z^2 \qquad C/cm^2 \qquad (33)$$

where τ is the beam duration in sec. Relativistic electron beams for fusion should, and as proposed[1] do, satisfy this requirement.

It should be mentioned that the assumption of poor magnetic field penetration leading to the creation of large electric fields in the plasma is not usually valid in present-day, low-temperature blow-off experiments. Comparing the electromagnetic skin depth[9] with the blow-off thickness[2] suggests that magnetic neutralization occurs when

$$\tau > 5 \times 10^{-5} Z \theta^{-5/2} \qquad sec. \qquad (34)$$

Thus, temperatures in excess of about 10 eV are required for substantial return currents.

In summary, it has been shown that the existence of large return-current-generated electric fields in pellet blow-off plasmas do not increase beam-energy deposition because they tend to inhibit beam penetration to high plasma densities.

ACKNOWLEDGEMENT

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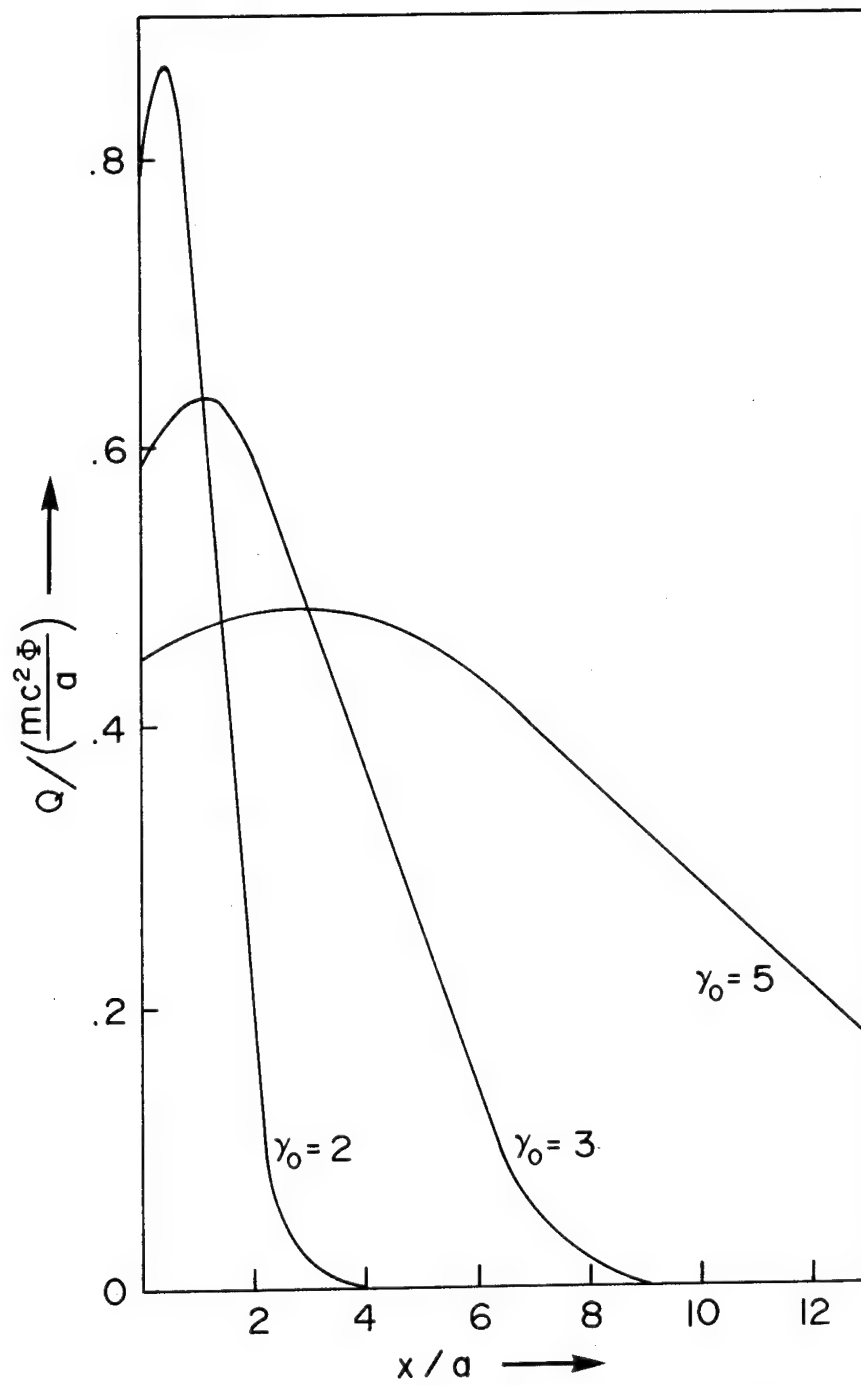
Fig. 1 - Normalized volume-heating rate vs depth into plasma
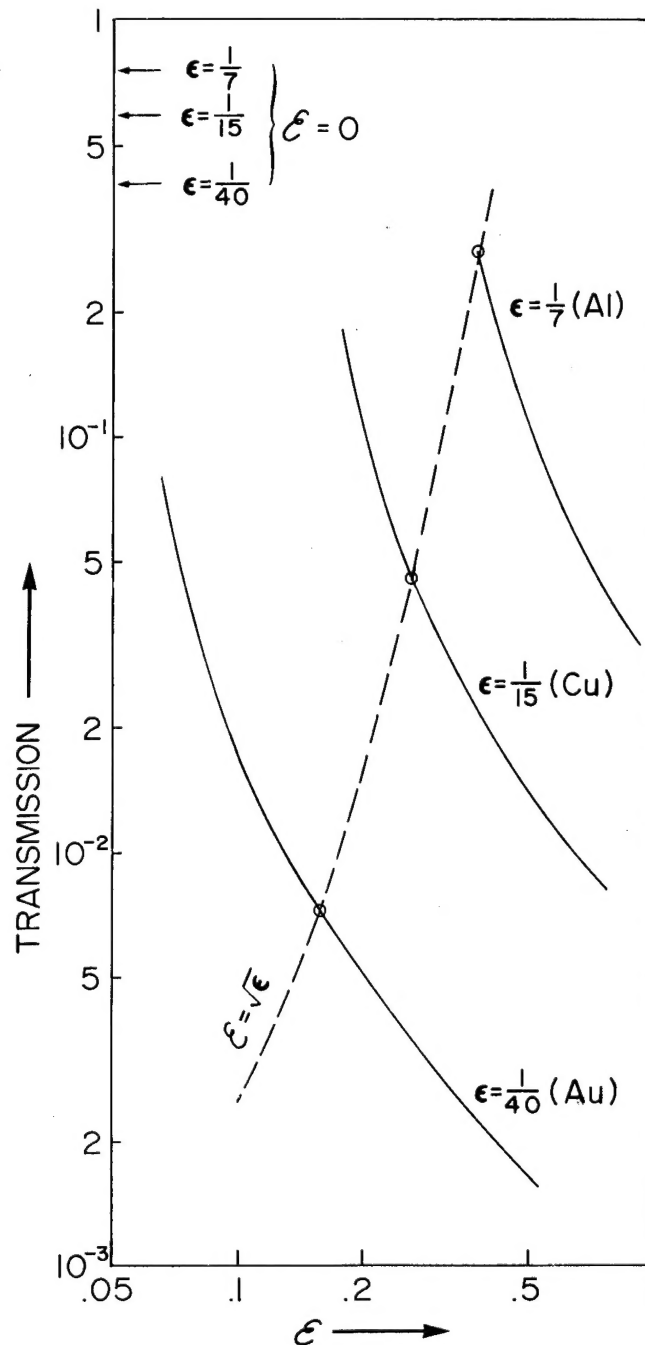for three values of γ_0

Fig. 2 - Transmission coefficient vs normalized electric field strength in plasma for $\gamma_o = 3$ and three atomic numbers. Return-current heating dominates to the right of the $\mathcal{E} = \epsilon^{1/2}$ curve.

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